Annals of Finance (2008) 4:131–155
DOI 10.1007/s10436-007-0073-3

RESEARCH ARTICLE

The social value of risk-free government debt

Stacey L. Schreft · Bruce D. Smith

Received: 1 November 2006 / Revised: 30 March 2007 / Published online: 16 May 2007

Abstract Can eliminating the stock of government bonds reduce welfare? When the government must raise revenue to pay interest on its bonds, the social value of government debt hinges on whether the benefits from greater portfolio diversification outweigh the costs of the revenue-raising efforts. A positive stock of debt is optimal only if interest payments are financed by an inflation tax, agents are not too risk averse, there is a government budget deficit, and the economy is on the bad side of the Laffer curve. Welfare is higher on the good side of the Laffer curve without bonds.

Keywords Government debt · Fiscal policy · Monetary policy · Portfolio allocation

JEL Classification Numbers E0 · E4 · E5 · E6 · H6 · G1

1 Introduction

Until relatively late in 2001, many projections had the stock of U.S. government debt outstanding shrinking dramatically—possibly going to zero—over the ensuing decade.

This paper was finished after Bruce Smith's death. The authors thank Kevin Huang, Jim Bullard, Mark Gertler, and Antoine Martin for helpful comments. Aarti Singh provided exceptional research support. The views expressed in this paper are not necessarily those of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

S. L. Schreft (✉)
Economic Research Department, Federal Reserve Bank of Kansas City, Kansas City, MO 64198, USA
e-mail: Stacey.L.Schreft@kc.frb.org

B. D. Smith
Department of Economics, University of Texas-Austin, Austin, TX 78712, USA

The projected debt reductions were a consequence of sustained projected government budget surpluses.

The possibility that the stock of government debt outstanding might become extremely small raised concerns on Wall Street and within the federal government. A primary concern was that an absence of government debt would deprive savers of an essentially safe asset. According to the argument, the complete, or near complete, elimination of government debt would force at least some economic actors to bear increased portfolio risk, thereby making them worse off.[1]

While this argument received considerable attention in informal discussions, there appears to have been no attempt to evaluate it in a formal economic model. This paper aims to fill this gap. It addresses the optimal level of a stock of risk-free government bonds in a monetary economy, described in Sects. 2–4. In that economy, currency coexists with a risky real asset that dominates it in rate of return. The economy is also one in which the Friedman rule is not optimal, as shown in Sect. 5, because it raises the rate of return on currency, resulting in insufficient investment in the real asset. By implication, if it is optimal for the government to issue bonds, then it is optimal for those bonds to pay a positive rate of interest.

With government bonds paying interest, a primary factor in the analysis is how the government finances its interest obligations on the debt. If the government could issue debt at no cost, clearly there would be a social benefit to its doing so because the debt would provide superior access to insurance. However, the real return on government debt is an endogenous variable, and if the government issues debt, it must also raise additional revenue to cover its interest obligations. This paper considers two scenarios with respect to how the government raises the necessary revenue. In the first, presented in Sect. 6, the government pays interest on the debt with seigniorage revenue. In the second, analyzed in Sect. 7, the government raises the necessary revenue through nondistortionary (i.e., lump-sum) taxation.

The analysis indicates that there is little case to be made for issuing government debt solely to provide agents with a risk-free asset. In particular, when interest payments on the debt are monetized, there is no benefit in terms of steady-state welfare to having a positive stock of government debt if there is a primary government budget surplus. In fact, issuing positive amounts of debt reduces steady-state utility. This result is important in the appropriate context because projections of a shrinking stock of U.S. government debt were based on projections that the budget would be in surplus. The findings suggest that, in such a scenario, there would be no argument in favor of having a stock of government debt outstanding.

If, on the other hand, there is a primary government budget deficit, then having safe government debt outstanding can raise steady-state welfare if agents are not too risk averse and if the economy is on the wrong side of the Laffer curve. In other words, it is optimal to have a positive stock of safe government debt if and only if

[1] There were two other concerns that received considerable attention as well. One was that without government debt, there would be no asset to serve as a benchmark against which risky assets could be priced. The other concern was over how the Federal Reserve would conduct monetary policy if there were not enough bonds available to conduct the desired open market operations. Schreft and Smith (2002) analyzes this second concern.

increases in the bond-to-money ratio reduce the equilibrium inflation rate. This is the case on the wrong side of the Laffer curve because unpleasant monetarist arithmetic (Sargent and Wallace 1981) does not apply there. But in that case, agents would be better off at the feasible equilibrium with a lower inflation rate.

Situations are also considered in which interest payments on government debt can be financed through nondistortionary taxation. It is shown that positive stocks of government debt are never optimal if the real return on government debt exceeds the economy's real rate of growth.

The framework for examining these issues is an economy with three assets: money, government bonds, and a risky storage technology. In this economy, as in Townsend (1980, 1987), spatial separation and limited communication create a transactions role for currency, despite its relatively low rate of return. Additionally, random movements of agents between locations create the analog of a liquidity-preference shock. Agents will want to be insured against such shocks. As in Diamond and Dybvig (1983), such insurance can be provided through intermediaries. The presence of intermediaries seems a reasonable feature of any analysis of the potential benefits of government debt since most holdings of government bonds are intermediated.

The potential benefits of government debt have been studied from various perspectives. One line of the literature, close in spirit to this article, considers the optimal financing of an exogenous government deficit in environments that restrict the policy instruments available (e.g., no lump-sum taxes) or where the Friedman rule is not optimal. See Smith and Villamil (1998), Villamil (1988), and Cavalcanti and Villamil (2003). Another line of literature considers the possibility that bonds allow the government to smooth taxes over time, but this literature typically works with nonmonetary economies. See Lucas and Stokey (1983), Barro (1997), Chari and Kehoe (1999), and Hall and Krieger (2000). Romer (1993) is related to this second line of literature in that he finds government bonds add value by allowing the government to raise revenue without relying as heavily on distorting income taxation. He assumes that using bonds or capital involves a positive, finite transaction cost, while money is costless to use. The assumption of differing transaction costs is comparable to the assumptions here that give rise to differences in liquidity.[2]

There are, of course, other motives for having a stock of government bonds outstanding beyond those discussed in the aforementioned literature. One motive is to ease borrowing or liquidity constraints.[3] Additionally, bonds can be valuable because they allow agents to achieve a better intertemporal allocation of resources than they could otherwise achieve. The model presented here abstracts from everything that would give rise to an alternative motive for maintaining a stock of interest-bearing

[2] This paper is like Romer's in several respects. Both papers allow for three assets to coexist, which is richer than most models. Both use an overlapping-generations model, compare steady-state equilibria, and focus on the lifetime utility of a representative young agent as the measure of welfare, ignoring the utility of the initial old generation. It is well known that in an overlapping generations model, steady states with different stocks of bonds would be Pareto noncomparable if welfare were evaluated by the Pareto criterion.

[3] This line of literature includes Aiyagari and McGrattan (1998), Woodford (1990), Aiyagari and Gertler (1991), and Kocherlakota (2003). There are no borrowing constraints in the environment studied here. In fact, here no agent other than the government could conceivably want to borrow. Indeed, under appropriate assumptions, markets are complete, although frictions exist that motivate monetary exchange.

government debt. Here, the goal is to focus the analysis on the benefits from greater portfolio diversification in the face of risky assets.

The major feature distinguishing this paper from the other papers reviewed here, including Romer's, is that one of the assets here—storage—is risky and as illiquid as the model's risk-free government bonds. The model thus allows for the possibility that agents will want to hold bonds to insure against the risk of a low return on storage and to balance that risk with the risk of relocation. There are no risky assets in the other papers in the literature, so those papers are silent on whether government debt can have value solely because of its risk-free nature.

2 The environment

The economy exists at discrete dates $t = 1, 2, \ldots$ and consists of two islands. Each island is populated by an identical infinite sequence of two-period-lived overlapping generations. At each date, a new young generation, consisting of a continuum of ex ante identical agents with unit mass, appears on each island. For simplicity, there is no population growth.

There is a single consumption good in the economy. Each agent born at dates $t \geqslant 1$ is endowed with $\omega > 0$ units of this good when young and nothing when old. Agents are assumed to consume only when old.[4] Letting c_t denote the old-age consumption of agents born at t, these agents have lifetime utility of $u(c_t) = c_t^{1-\rho} \Big/ (1-\rho)$, $\rho > 0$.

There are three primary assets in the economy. One is a linear storage technology. A unit of consumption invested in this technology at t yields x units of consumption at $t+1$. The variable x is random and realized at $t+1$. Each period it is drawn from a distribution F, with probability density function f. The function f has support $\left[\underline{x}, \bar{x}\right]$. Since all investments in storage at t yield the same random return, there is aggregate randomness governing the return on storage investments. Once x is realized, it is observable by all agents. The mean of x is $\hat{x} > 1$, so in an expected value sense, investments in the storage technology are socially efficient.

In addition to storage, there is a stock of one-period government bonds. This stock may be positive, negative, or zero. Government bonds pay a certain gross real return of R_t from t to $t+1$. Thus, in keeping with the discussion of the importance of government debt in providing a safe, real return, there is no randomness to the payoff on bonds. B_t denotes the nominal per capita stock of government debt outstanding at t and is a choice variable of the government.

The third primary asset in the economy is fiat money. M_t denotes the per capita stock of money outstanding at time t. p_t denotes the price level at time t. As will become apparent, these variables display no randomness in a perfect-foresight equilibrium. In what follows, the per capita stock of real money balances is denoted $z_t \equiv M_t \big/ p_t$, while the real value of debt outstanding is denoted $b_t \equiv B_t \big/ p_t$.

[4] This assumption is not important to the analysis. It merely simplifies the consumption-saving decision of the young.

2.1 The government

The government is assumed throughout to have a constant nonnegative per capita real expenditure level of g. In addition, it levies a real, time-independent lump-sum tax of τ on all young agents at all dates. The government then faces the following budget constraint for $t > 1$:

$$g - \tau + R_{t-1}b_{t-1} = b_t + z_t - z_{t-1}\left(\frac{p_{t-1}}{p_t}\right). \tag{1}$$

Government policy consists of a choice of two variables. First, following standard specifications in a variety of monetary models (e.g., Schreft and Smith 2000, 2002), the government sets once and for all a value for the bond-to-money ratio. That value is $\mu_t \equiv b_t / z_t$. This specification is not central to the analysis since the primary concern here is simply to establish when it is optimal to have a positive stock of government debt. Clearly, the stock of government debt outstanding is zero if $\mu_t = 0$.

In addition to setting a value for μ, the government confronts some choices about monetary and fiscal policy. Two scenarios are considered below. In the first scenario, the money supply grows according to $M_{t+1} = \sigma M_t$, with σ set once and for all at $t = 1$ and M_0, the initial money supply, exogenously given and held by the old at $t = 0$. This scenario leaves tax collections τ to be determined endogenously. In the second scenario, τ is exogenously given. This renders the money growth rate an endogenous variable.

The reason for considering two alternative scenarios for government policy is as follows. As noted in the introduction, when government debt with a safe real return is introduced, interest on this debt must be paid in some way. With money (tax) financing, an increase in government interest obligations associated with a change in μ_t must be financed by changes in seigniorage revenue (taxation). The desirability of having the government issue a positive stock of debt with a safe real return potentially depends on how interest obligations on the debt are financed. Thus, each scenario is considered below.

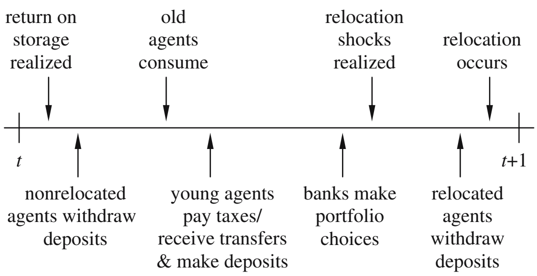
2.2 Spatial separation and limited communication

The reason for introducing two islands is to create a role for money in transactions and a role for banks. This section describes the nature of the economy's spatial separation and limited communication.

In the first period of life, each agent is assigned to one of the two locations. Trade and communication can occur only among agents who are in the same location at the same time. In particular, cross-location trade or communication is not possible.

When young, agents must allocate their savings, either directly or through intermediaries, among the economy's three primary assets. After this portfolio allocation occurs, a fraction π of young agents is randomly selected to move to the other location, with π being common across locations. The value π is known at the beginning of each

Fig. 1 The timing of events



period, but the specific identities of the agents to be relocated are realized only after portfolio allocation and consumption have occurred at t.

The significance of stochastic relocation is as follows. First, storage investments made at t do not mature until $t + 1$, and goods invested in storage cannot be transported across locations. In addition, bonds are assumed to not be usable in interlocation exchange. This could occur because government debt is issued in denominations that are too large to be used by individuals in transactions. In any event, the objective of the analysis is to determine whether it is optimal for the government to issue liabilities with a safe real return that exceeds the rate of return on money. Doing so is only possible if the government limits the potential for bonds to be used in transactions, as is assumed to be the case.

The observations of the previous paragraph imply that agents who are relocated do not carry bonds or storage investments with them. In addition, the assumption about the lack of interlocation communication implies that relocated agents cannot trade using checks, credit cards, or any liabilities that are claims against assets held in the other location. Thus, the event of being relocated forces agents to liquidate bonds and storage to obtain cash, which can be used in transactions in the other location. As in Townsend (1980, 1987), spatial separation and limited communication create a role for currency as a transactions medium for agents who are relocated.

This need to liquidate assets if an agent is relocated represents a type of liquidity preference shock against which agents will wish to be insured. This insurance can be provided by banks, as in Diamond and Dybvig (1983). The behavior of these banks is described in the next section.

Finally, after agents are relocated, period t ends. At $t+1$ storage returns are realized, and agents who are not relocated can consume out of the returns on any asset. Agents who are relocated use their cash balances to purchase consumption. The timing of events is depicted in Fig. 1.

3 Bank behavior

As in Diamond and Dybvig (1983), the structure of the model makes it natural to think of banks at t as coalitions of ex ante identical young agents. These agents deposit their entire after-tax endowment, $\omega - \tau$, with a bank.[5] The bank then chooses a gross real

[5] The assumptions on the timing of exchange imply that banks cannot usefully transfer resources between members of different generations.

return, d_t^m, to pay to agents who are relocated between t and $t+1$, and a return schedule $d_t(x_{t+1})$ to be paid at $t+1$ to agents who did not relocate. The latter repayment necessarily depends on the returns on the bank's storage investments. However, since these returns are not realized until $t+1$, the payments made to relocated agents cannot depend on the realization of x. In addition, the bank chooses values m_t, $\tilde{b}_t$, and s_t for the real value of money balances, government bonds, and storage investments that it holds. These choices must obviously satisfy the bank's balance sheet constraint:

$$m_t + \tilde{b}_t + s_t \leqslant \omega - \tau. \tag{2}$$

In making these choices, a bank views itself as unable to affect the returns on storage investments. In addition, the bank is assumed to act competitively in asset markets. That is, it takes the return on real balances, p_t / p_{t+1}, and the return on bonds, R_t, as unaffected by its own choices. Finally, each bank views itself as being able to trade bonds with a safe real return, even if the outstanding supply of government debt is zero.

If I_t denotes the gross nominal rate of interest on debt, then $I_t \equiv R_t\left(p_{t+1}/p_t\right)$. Throughout the analysis, it is assumed that $R_t \geqslant p_t / p_{t+1}$, which is equivalent to the assumption that $I_t \geqslant 1$, so that the nominal interest rate on bonds is positive.[6] When $I > 1$, banks will hold cash reserves only to make payments to relocated agents, given that withdrawal demand is predictable. They will carry no cash reserves across periods. In contrast, when $I = 1$, some reserves may be carried across periods. Consequently, in specifying the bank's problem, it is useful to let $\tilde{m}_t$ be money held by banks to pay movers and $\hat{m}_t$ be money held across periods to pay nonmovers, with $m_t = \tilde{m}_t + \hat{m}_t$.

Given this observation, a bank faces the following additional constraints. First, it must hold enough cash to honor its promised payments to agents who are relocated:

$$\pi d_t^m(\omega - \tau) \leqslant \tilde{m}_t \frac{p_t}{p_{t+1}}. \tag{3}$$

In addition, promised repayments to nonrelocated agents cannot exceed the value of the bank's other assets. That is,

$$(1-\pi) d_t(x_{t+1})(\omega - \tau) \leqslant \hat{m}_t \frac{p_t}{p_{t+1}} + R_t \tilde{b}_t + x_{t+1} s_t. \tag{4}$$

The bank chooses its returns and its portfolio to maximize the expected utility of a representative depositor,

$$\left[\frac{(\omega-\tau)^{1-\rho}}{1-\rho}\right]\left\{\pi\left(d_t^m\right)^{1-\rho} + (1-\pi)\left[\int_{\underline{x}}^{\bar{x}} (d_t(x))^{1-\rho} f(x)\mathrm{d}x\right]\right\},$$

subject to (2)–(4).

[6] As will be shown in Sect. 5 below, it is optimal to have $I > 1$.

Alternatively, since (2)–(4) must hold with equality in an optimum, the bank's problem can be written slightly differently if $\tilde{\gamma}_t^m \equiv \tilde{m}_t \big/ (\omega - \tau)$ denotes the fraction of deposits held as cash to pay movers, $\hat{\gamma}_t^m \equiv \hat{m}_t \big/ (\omega - \tau)$ denotes the fraction of deposits held across periods as cash to pay nonmovers, and $\gamma_t^b \equiv \tilde{b} \big/ (\omega - \tau)$ denotes the fraction of deposits held in the form of government debt. Then the bank can be viewed as choosing $\tilde{\gamma}_t^m$, $\hat{\gamma}_t^m$, and γ_t^b to maximize

$$\left[\frac{(\omega-\tau)^{1-\rho}}{1-\rho}\right]\left\{\pi\left(\frac{\tilde{\gamma}_t^m\left(p_t/p_{t+1}\right)}{\pi}\right)^{1-\rho}\right.$$
$$\left.+(1-\pi)\int_{\underline{x}}^{\bar{x}}\left[\frac{\hat{\gamma}_t^m\left(p_t/p_{t+1}\right)+R_t\gamma_t^b+x\left(1-\tilde{\gamma}_t^m-\hat{\gamma}_t^m-\gamma_t^b\right)}{1-\pi}\right]^{1-\rho} f(x)\mathrm{d}x\right\}.$$

In this optimization problem, the bank can choose either positive or negative debt levels. The bank's optimal behavior is then described by the following first-order conditions for $\tilde{\gamma}_t^m$, $\hat{\gamma}_t^m$, and γ_t^b, respectively:

$$\pi^\rho\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\tilde{\gamma}_t^m\right)^{-\rho}$$
$$=(1-\pi)^\rho\int_{\underline{x}}^{\bar{x}} x\left[\hat{\gamma}_t^m\frac{p_t}{p_{t+1}}+R_t\gamma_t^b+x\left(1-\tilde{\gamma}_t^m-\hat{\gamma}_t^m-\gamma_t^b\right)\right]^{-\rho} f(x)\mathrm{d}x, \quad (5)$$

$$\int_{\underline{x}}^{\bar{x}}\left(\frac{p_t}{p_{t+1}}-x\right)\left[\hat{\gamma}_t^m\frac{p_t}{p_{t+1}}+R_t\gamma_t^b+x\left(1-\tilde{\gamma}_t^m-\hat{\gamma}_t^m-\gamma_t^b\right)\right]^{-\rho} f(x)\mathrm{d}x \leqslant 0, \quad (6)$$

$$\int_{\underline{x}}^{\bar{x}}\left(R_t-x\right)\left[\hat{\gamma}_t^m\frac{p_t}{p_{t+1}}+R_t\gamma_t^b+x\left(1-\tilde{\gamma}_t^m-\hat{\gamma}_t^m-\gamma_t^b\right)\right]^{-\rho} f(x)\mathrm{d}x \leqslant 0. \quad (7)$$

4 General equilibrium

In any general equilibrium, the money and bond markets must clear. Since all beginning-of-period demand for money balances derives from banks' demands for cash reserves, the money market clears at t if

$$(\omega-\tau)\left(\tilde{\gamma}_t^m+\hat{\gamma}_t^m\right)=z_t\equiv\frac{M_t}{p_t}. \quad (8)$$

Because $\tilde{\gamma}_t^m$, $\hat{\gamma}_t^m$, and τ_t are chosen before the relevant return on storage is realized, Eq. (8) implies that p_t is not stochastic. In addition, since the demand for and the supply of government debt must be equal,

$$(\omega - \tau)\gamma_t^b = b_t \equiv \frac{B_t}{p_t}. \tag{9}$$

It follows that the stance of monetary policy, μ_t, is $b_t/z_t = \gamma_t^b/(\tilde{\gamma}_t^m + \hat{\gamma}_t^m)$. Hereafter, attention is confined to steady-state equilibria so time subscripts are omitted.

Using (8) and (9), the government budget constraint (1) in a steady state becomes

$$(R-1)\gamma^b = \left(1 - \frac{p_t}{p_{t+1}}\right)(\tilde{\gamma}^m + \hat{\gamma}^m) - \frac{g-\tau}{\omega-\tau}. \tag{10}$$

In addition, the first-order conditions (Eqs. (5)–(7)) become

$$\pi^\rho \left(\frac{p_t}{p_{t+1}}\right)^{1-\rho} (\tilde{\gamma}^m)^{-\rho}$$
$$= (1-\pi)^\rho \int_{\underline{x}}^{\bar{x}} x\left[\hat{\gamma}^m \frac{p_t}{p_{t+1}} + R\gamma^b + x\left(1 - \tilde{\gamma}^m - \hat{\gamma}^m - \gamma^b\right)\right]^{-\rho} f(x)\mathrm{d}x, \tag{11}$$

$$\int_{\underline{x}}^{\bar{x}} \left(\frac{p_t}{p_{t+1}} - x\right)\left[\hat{\gamma}^m \frac{p_t}{p_{t+1}} + R\gamma^b + x\left(1 - \tilde{\gamma}^m - \hat{\gamma}^m - \gamma^b\right)\right]^{-\rho} f(x)\mathrm{d}x \leqslant 0, \tag{12}$$

$$\int_{\underline{x}}^{\bar{x}} (R - x)\left[\hat{\gamma}^m \frac{p_t}{p_{t+1}} + R\gamma^b + x\left(1 - \tilde{\gamma}^m - \hat{\gamma}^m - \gamma^b\right)\right]^{-\rho} f(x)\mathrm{d}x \leqslant 0. \tag{13}$$

Given a specification of government policy ($b = \mu z$ and a choice of either money or tax financing), Eqs. (10)–(13) are the steady-state equilibrium conditions.

The remainder of this paper analyzes the steady states of this economy. The analysis begins by considering the optimality of the Friedman rule. If the Friedman rule is optimal, as it is in most monetary models, then the government cannot improve welfare by introducing a stock of risk-free, interest-bearing government debt. That is, any government debt issued should be indistinguishable from currency in its rate of return. As the next section shows, in the economy studied here, the Friedman rule does not hold.

5 Steady states with $I = 1$

The Friedman rule recommends conducting monetary policy to achieve an equilibrium with $I = 1$. Under the Friedman rule, government bonds play no role in the economy for two reasons. First, bonds pay the same rate of return as currency but are less liquid because they cannot be carried across islands. Second, storage is as illiquid as government bonds, but dominates bonds in rate of return. Thus, an equilibrium in which monetary policy achieves the Friedman rule has $\mu = 0$ and $\gamma^b = 0$, and since $I = 1$, the real interest rate satisfies $R = p_t/p_{t+1} = 1/\sigma$. It follows that the optimality of the Friedman rule can be considered by setting $\mu = 0$ and evaluating whether welfare increases as μ increases.

Following the discussion in Sect. 3, steady-state welfare in an economy in which the Friedman rule is achieved is given by the expression

$$W \equiv \frac{(\omega - \tau)^{1-\rho}}{1-\rho} \left\{ \pi \left[\left(\frac{\tilde{\gamma}^m}{\pi} \right) \left(\frac{p_t}{p_{t+1}} \right) \right]^{1-\rho} + (1-\pi) \int_{\underline{x}}^{\bar{x}} \left[\frac{\hat{\gamma}^m \frac{p_t}{p_{t+1}} + R\gamma^b + x\left(1-\tilde{\gamma}^m-\hat{\gamma}^m - \gamma^b\right)}{1-\pi} \right]^{1-\rho} f(x)\mathrm{d}x \right\}. \quad (14)$$

With $\mu = 0 = \gamma^b$ and $R = p_t/p_{t+1}$, W is only a function of I, and the Friedman rule is not optimal if $W'(I)|_{I=1} > 0$.

Equilibrium is indeterminate with $I = 1$ because $\hat{\gamma}^m$ is indeterminate. But certainly a possible steady state is the limit of the one with $I > 1$ as $I \to 1$. Any other equilibrium with $I = 1$ involves banks holding even more excess reserves, fewer government bonds, and less storage, making the benefit from having interest-bearing and risk-free government bonds even lower. Hence, nothing is lost by considering the steady state of an economy with $I > 1$ in the limit, as $I \to 1$.

In this limit economy, $\hat{\gamma}^m = 0$. Since $\mu = 0$, which implies $\gamma^b = 0$, (13) reduces to

$$R = \frac{\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x}{\int_{\underline{x}}^{\bar{x}} x^{-\rho} f(x)\mathrm{d}x} < \hat{x}. \quad (15)$$

This implies that the safe real rate of return is less than the expected return to storage.

Since $\hat{\gamma}^m = 0$ in the limit economy, $\tilde{\gamma}^m$ will be denoted without the tilde henceforth for notational simplicity. The characterization of steady states with $\mu = 0$ can then be completed by setting $\hat{\gamma}^m = \gamma^b = 0$ in (11) and solving for γ^m to obtain

$$\gamma^m = \left\{ 1 + \frac{1-\pi}{\pi} \left[\int_{\underline{x}}^{\bar{x}} \left(x \frac{p_{t+1}}{p_t} \right)^{1-\rho} f(x)\mathrm{d}x \right]^{\frac{1}{\rho}} \right\}^{-1}. \quad (16)$$

This can be rewritten as

$$\gamma^m(I) = \left\{ 1 + \frac{1-\pi}{\pi} I^{\frac{1-\rho}{\rho}} \left[\int_{\underline{x}}^{\bar{x}} \left(\frac{R}{x} \right)^{\rho} f(x)\mathrm{d}x \right]^{\frac{1}{\rho}} \right\}^{-1} \quad (17)$$

so that γ^m is explicitly a function of I.

In addition, the government budget constraint (10) reduces to

$$\left(1-\frac{p_t}{p_{t+1}}\right)\gamma^m(I)=\frac{g-\tau}{\omega-\tau}, \tag{18}$$

indicating that any deficit must be financed through seigniorage and that there are no debt obligations to honor since there are no bonds. Solving (18) for $\omega-\tau$ yields

$$\omega-\tau=\frac{\omega-g}{1-\left(\frac{I-R}{I}\right)\gamma^m(I)}\equiv\Omega(I).$$

This allows steady-state welfare, (14), to be written as

$$W(I)\equiv\frac{\Omega(I)^{1-\rho}}{1-\rho}\Gamma(I),$$

where

$$\Gamma(I)\equiv\pi\left[\frac{\gamma^m(I)}{\pi}\frac{R}{I}\right]^{1-\rho}+(1-\pi)\int_{\underline{x}}^{\bar{x}}\left(\frac{x\left[1-\gamma^m(I)\right]}{1-\pi}\right)^{1-\rho}f(x)\mathrm{d}x.$$

Given $\rho<1$, the condition for the nonoptimality of the Friedman rule, $W'(I)|_{I=1}>0$, can be written

$$\frac{I\Omega'}{\Omega}>-\frac{1}{1-\rho}\frac{\Gamma' I}{\Gamma}. \tag{19}$$

Equation (19) indicates that the optimality of the Friedman rule depends essentially on the relative magnitudes of the interest elasticities of Ω and Γ. The elasticity on the left-hand side is

$$\left[\frac{\gamma^m(1)}{1+(R-1)\gamma^m(1)}\right]\left(R+\left(\frac{1-\rho}{\rho}\right)(R-1)\left[1-\gamma^m(1)\right]\right),$$

and the one on the right is

$$\frac{\pi^\rho\left[\gamma^m(1)R\right]^{1-\rho}}{\pi^\rho\left[\gamma^m(1)R\right]^{1-\rho}+(1-\pi)^\rho\int_{\underline{x}}^{\bar{x}}x\left[1-\gamma^m(1)\right]^{1-\rho}f(x)\mathrm{d}x}.$$

After some algebra, (19) reduces to

$$\gamma^m(1)\left\{\frac{R\gamma^m(1)+\left[1-\gamma^m(1)\right]\left[1+\frac{R-1}{\rho}\right]}{R\gamma^m(1)+\left[1-\gamma^m(1)\right]}\right\}>\gamma^m(1).$$

If $R > 1$, then the inequality in (19) is satisfied, and the Friedman rule is suboptimal, as in Schreft and Smith (2002) and Paal and Smith (2000), because it makes currency too good an asset. In an equilibrium with $I = 1$, banks hold too small a share of their portfolios in storage and too large a share in currency compared to what is optimal. In contrast, if $R \leqslant 1$, the safe real rate of return is less than the economy's growth rate, and the Friedman rule is optimal. But if the Friedman Rule is optimal, then the introduction of risk-free government bonds will not raise steady-state welfare. Hence, the remainder of this paper considers only equilibria in which all $R > 1$, so the Friedman rule is suboptimal and the introduction of interest-bearing government liabilities can be beneficial.

6 Steady states with $I > 1$ and money financing

This section and the next consider how the size of the stock of government debt outstanding affects steady-state welfare in equilibria with $I > 1$. As noted in the introduction, a central issue concerns how the government raises the revenue needed to cover its interest obligations on its debt. This section considers the scenario in which taxation remains fixed as μ is varied. By implication, the government must finance its interest payments with seigniorage revenue. This section starts with a local analysis, considering the welfare implications of introducing a stock of risk-free government bonds into an economy that has no government bonds outstanding (that is, one with $\mu = 0$). General results are derived from the local analysis. The section proceeds to consider the implications of introducing government bonds into an economy in which some bonds are already present. Because of the complexity of this problem, numerical methods are used.

6.1 Introducing bonds into the economy

As already stated, in equilibria with $I > 1$, $\hat{\gamma}^m = 0$. Substituting (17) into (18) yields the government budget constraint with money financing of interest obligations $\mu = 0$:

$$\left(1 - \left(\frac{p_{t+1}}{p_t}\right)^{-1}\right)\left(1 + \left(\frac{1-\pi}{\pi}\right)\left(\frac{p_{t+1}}{p_t}\right)^{\frac{1-\rho}{\rho}}\left[\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x\right]^{\frac{1}{\rho}}\right)^{-1}$$
$$= \frac{g-\tau}{\omega-\tau}. \tag{20}$$

The left side of (20) is seigniorage revenue. If $\rho < 1$, then (20) describes a relatively conventional seigniorage Laffer curve, as depicted in Fig. 2. If $\rho > 1$, then the seigniorage Laffer curve is always upward sloping; there is no bad side of the curve.

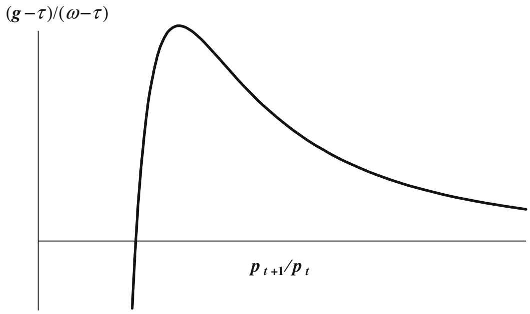


Fig. 2 The Seigniorage Laffer curve

Seigniorage revenue is maximized at the inflation rate, denoted $\left(p_{t+1}\big/p_t\right)^*$, that satisfies

$$1+\left(\frac{1}{\rho}\right)\left(\frac{1-\pi}{\pi}\right)\left(\frac{p_{t+1}}{p_t}\right)^{\frac{1-\rho}{\rho}}\left[\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x\right]^{\frac{1}{\rho}}$$
$$-\left(\frac{1-\rho}{\rho}\right)\left(\frac{1-\pi}{\pi}\right)\left(\frac{p_{t+1}}{p_t}\right)^{\frac{1}{\rho}}\left[\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x\right]^{\frac{1}{\rho}}=0. \tag{21}$$

It follows that a steady-state equilibrium exists [that is, (20) has nonnegative solutions for $p_{t+1}\big/p_t$, given $(g-\tau)\big/(\omega-\tau)$] if and only if the government's primary budget deficit does not exceed the maximum revenue that can be raised from seigniorage:

$$\left(1-\left(\left(\frac{p_{t+1}}{p_t}\right)^*\right)^{-1}\right)\left(1+\left(\frac{1-\pi}{\pi}\right)\left(\left(\frac{p_{t+1}}{p_t}\right)^*\right)^{\frac{1-\rho}{\rho}}\left[\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x\right]^{\frac{1}{\rho}}\right)^{-1}$$
$$\geqslant \frac{g-\tau}{\omega-\tau}. \tag{22}$$

Equation (22) is assumed to be satisfied. It follows that (20) has at least one solution. More specifically, if (22) is satisfied as a strict inequality and if $\rho < 1$, then (20) has two solutions. The solution with a low (high) inflation rate lies on the "good" ("bad") side of the Laffer curve. As Fig. 2 indicates, higher values of g, given τ, lead to higher (lower) steady-state rates of inflation on the good (bad) side of the Laffer curve. In contrast, if $\rho > 1$, the Laffer curve increases at a decreasing rate, and there is just one solution, the one corresponding qualitatively to the solution on the good side of the curve when $\rho < 1$.

Following the discussion in Sect. 5, steady-state welfare is given by (14). With money financing, not only are R, $\tilde{\gamma}^m$, and $\hat{\gamma}^m$ functions of μ, but $p_t\big/p_{t+1}$ is as well. These functions are implicitly defined by Eqs. (10)–(13). It remains to evaluate $W'(\mu)|_{\mu=0}$. When this derivative is positive, steady-state welfare will be increased if the government introduces some safe government debt, though perhaps only a small amount.

With τ fixed, the envelope theorem implies that

$$\frac{W'(\mu)}{(\omega-\tau)^{1-\rho}} = \pi^{\rho}\left(\gamma^{m}\right)^{1-\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\frac{\partial\left(\frac{p_t}{p_{t+1}}\right)}{\partial\mu}$$
$$+(1-\pi)^{\rho}\gamma^{b}\left(\frac{\partial R}{\partial\mu}\right)\int_{\underline{x}}^{\bar{x}}\left[\frac{R\gamma^{b}+x\left(1-\gamma^{m}-\gamma^{b}\right)}{1-\pi}\right]^{-\rho} f(x)\mathrm{d}x.$$

Thus, since $\gamma^{b}=\mu\gamma^{m}$,

$$\frac{W'(0)}{(\omega-\tau)^{1-\rho}} = \pi^{\rho}\left(\gamma^{m}\right)^{1-\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}, \tag{23}$$

with γ^{m} given by (16) and p_t/p_{t+1} given by (20). Moreover, as is clear from (23), $W'(0)$ has the same sign as $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}$. Thus, as μ is increased slightly above zero, the effect on steady-state welfare depends on the effect of higher values of μ on p_t/p_{t+1}.

To analyze the social value of risk-free government debt when interest obligations on that debt are monetized, it remains to determine the sign of $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}$. The appendix shows that $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}$ and $\left(\partial\gamma^{m}/\partial\mu\right)\big|_{\mu=0}$ solve the two equations

$$\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0} = -(R-1)+\left(1-\frac{p_t}{p_{t+1}}\right)\left(\frac{1}{\gamma^{m}}\right)\left.\frac{\partial\gamma^{m}}{\partial\mu}\right|_{\mu=0} \tag{24}$$

and

$$\left(\frac{1-\rho}{\rho}\right)\left(\frac{p_t}{p_{t+1}}\right)^{-1}\gamma^{m}\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}$$
$$=\left.\frac{\partial\gamma^{m}}{\partial\mu}\right|_{\mu=0}\left\{1+\pi\left(\frac{p_t}{p_{t+1}}\right)^{\frac{1-\rho}{\rho}}\left[(1-\pi)^{\rho}\int_{\underline{x}}^{\bar{x}}x^{1-\rho}f(x)\mathrm{d}x\right]^{\frac{-1}{\rho}}\right\}, \tag{25}$$

and that the solution for $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}$ satisfies

$$-\left(\frac{1-\rho}{\rho}\right)\left(\frac{p_{t+1}}{p_t}\right)\gamma^m(R-1)=\frac{\partial\gamma^m}{\partial\mu}\bigg|_{\mu=0}\left(\frac{1}{\rho}\right)\left\{1-H\left(\frac{p_{t+1}}{p_t}\right)\right\},\qquad(26)$$

with

$$H\left(\frac{p_{t+1}}{p_t}\right)\equiv(1-\rho)\left(\frac{p_{t+1}}{p_t}\right)-\rho\pi\left(\frac{p_{t+1}}{p_t}\right)^{\frac{\rho-1}{\rho}}\left[(1-\pi)^{\rho}\int\limits_{\underline{x}}^{\bar{x}}x^{1-\rho}f(x)\mathrm{d}x\right]^{\frac{-1}{\rho}}.$$

Hence, the sign of $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}$ depends on the sign of $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}$, and the sign of $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}$ depends on the magnitude of ρ and the sign of H, given that attention is restricted to equilibria with $R>1$, as discussed in Sect. 5.

Some properties of the function H will prove useful in the subsequent analysis. These are stated in Lemma 1.

Lemma 1 *(a)* $H(1)<1$. *(b) When* $\rho<1$, $H'\left(p_{t+1}/p_t\right)>0$. *(c)* $H\left[\left(p_{t+1}/p_t\right)^*\right]=1$.

Lemma 1 follows directly from inspection of $H\left(p_{t+1}/p_t\right)$ and from the observation that the condition $H\left(p_{t+1}/p_t\right)=1$ is exactly condition (21) and therefore satisfied for $p_{t+1}/p_t=\left(p_{t+1}/p_t\right)^*$.

Since the sign of $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}$ depends on the magnitude of ρ as well as the sign of $H\left(p_{t+1}/p_t\right)$, it is instructive to consider two cases: $\rho<1$ and $\rho>1$.

Case 1 $\rho<1$. If $\rho<1$, then (25) implies that $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}$ has the same sign as $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}$. Thus, $W'(0)>0$ if and only if $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}>0$. Moreover, it is evident from (26) that in steady states with $R>1$, $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}>0$ iff $H\left(p_{t+1}/p_t\right)>1$. The effect of μ on steady-state welfare depends, then, on whether $H\left(p_{t+1}/p_t\right)>1$. Lemma 1 asserts that $H\left(p_{t+1}/p_t\right)>1$ iff $p_{t+1}/p_t>\left(p_{t+1}/p_t\right)^*$. This can transpire only if $g>\tau$ (that is, if there is a primary government budget deficit), and if the economy's equilibrium lies on the wrong side of the Laffer curve.

Case 2 $\rho>1$. When $\rho>1$, (25) implies that the derivatives $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}$ and $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}$ are opposite in sign. In addition, (26) implies that $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}>0$ in steady states with $R>1$ because $H\left(p_{t+1}/p_t\right)<0$ necessarily. Thus, $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}<0$ and consequently $W'(0)<0$. That is, when agents are relatively risk averse, there are no welfare gains from introducing a small stock of risk-free government debt into an economy.

The following propositions summarize these observations.

Proposition 1 *(a) If* $\rho<1$ *and* $g\leqslant\tau$, *then in steady states with* $R>1$, $\partial\gamma^m/\partial\mu\big|_{\mu=0}<0$, $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}<0$, *and thus* $W'(0)<0$.

(b) If $\rho < 1$ and $g > \tau$ in steady states with $R > 1$, then $\left(\partial \gamma^m / \partial \mu\right)\big|_{\mu=0} < 0$, $\left(\partial\left(p_t / p_{t+1}\right) / \partial \mu\right)\big|_{\mu=0} < 0$, and $W'(0) < 0$ when the economy's equilibrium rests on the good side of the Laffer curve. In contrast, on the bad side of the Laffer curve, $\left(\partial \gamma^m / \partial \mu\right)\big|_{\mu=0} > 0$, $\left(\partial\left(p_t / p_{t+1}\right) / \partial \mu\right)\big|_{\mu=0} > 0$, and $W'(0) > 0$.

Proposition 2 *If $\rho > 1$, then in steady states with $R > 1$, $\left(\partial \gamma^m / \partial \mu\right)\big|_{\mu=0} > 0$, $\left(\partial\left(p_t / p_{t+1}\right) / \partial \mu\right)\big|_{\mu=0} < 0$, and thus $W'(0) < 0$.*

Proposition 1(a) states a strong result. Introducing a small stock of interest-bearing, risk-free government bonds into an economy without bonds when $R > 1 > \rho$ necessarily reduces steady-state welfare unless there is a primary budget deficit. Thus, with budget surpluses, which imply that any debt issued will ultimately vanish, agents will not benefit from actions taken to maintain a small stock of debt if the interest payments on the debt will be financed with seigniorage.

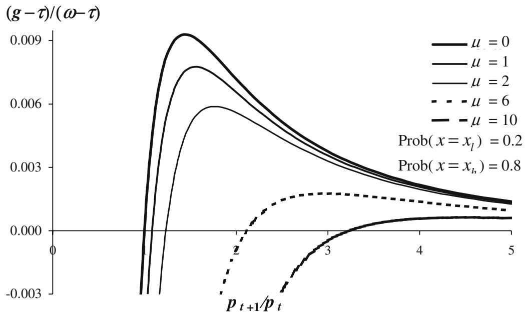
The existence of a primary budget deficit, however, does not necessarily imply that introducing a small stock of government debt will increase steady-state welfare. As Proposition 1(b) indicates, only in steady states on the bad side of the Laffer curve does an increase in the bond-to-money ratio raise the rate of return on currency and thus welfare. There unpleasant monetarist arithmetic (Sargent and Wallace 1981; or Bhattacharya et al. 1998) does not hold because the introduction of government debt has two beneficial consequences. It introduces a risk-free asset, providing insurance against the rate-of-return risk associated with the economy's real asset. It also reduces the inflation tax rate, lowering the cost of holding currency as insurance against the risk of relocation. Both consequences are welfare improving. In contrast, on the good side of the Laffer curve, unpleasant monetarist arithmetic does hold. The introduction of government debt gives agents beneficial insurance against rate-of-return risk in that region as well, but it also has the negative—and offsetting—effect of raising the inflation tax rate. The net effect is a reduction in welfare.

Proposition 2 simply reflects the fact that reserve demand is increasing in the nominal interest rate because the income effect dominates the substitution effect when $\rho > 1$ and $\mu = 0$. In this case, the Laffer curve is strictly upward sloping, so there is no bad side of the curve. Introducing a small stock of bonds into the economy thus reduces welfare.

6.2 Adding to an existing stock of bonds

The impact on steady-state welfare of increasing μ from any level, given $I > 1$ and thus $\hat{\gamma}^m = 0$, can be determined by fixing $(g - \tau) / (\omega - \tau)$ and solving (5)–(7) and (10) for γ^m, R, and p_{t+1} / p_t as functions of μ and using $\gamma^b = \mu \gamma^m$ to obtain γ^b. Since closed-form solutions for this system cannot be derived, numerical methods are used. As discussed in Sect. 6.1, not all values of $(g - \tau) / (\omega - \tau)$ can be financed in equilibrium. Consequently, the first step in solving the model numerically is to derive the Laffer curve under a particular parameterization. Attention can then be restricted to the effect on equilibrium portfolio allocations and welfare of increases in μ, given values of $(g - \tau) / (\omega - \tau)$ that are feasible for the parameterized economy.

Fig. 3 Seigniorage Laffer curves net of interest obligations for $\rho = 0.3, \pi = 0.1, x_l = 0.1, x_h = 1.56$



The numerical analysis was conducted for a wide range of the parameter space: $\pi \in (0, 1)$; $\rho > 0$; $f(x)$ is discrete with $x \in \{x_l, x_h\}$, $f(x_l) = q$, and $f(x_h) = 1 - q$ such that (15) is satisfied; and $(g - \tau)\big/(\omega - \tau)$ such that (22) is satisfied. All parameterizations with $R > 1$ yield the same qualitative results, depending only on whether $\rho > 1$ or $\rho < 1$, and attention remains restricted to equilibria with $R > 1$ as in previous sections. Thus, the results for just two parameterizations are presented here, one with $\rho < 1$ and one with $\rho > 1$. When $\rho < 1$, the substitution effect of a change in the nominal interest rate on the reserve–deposit ratio dominates the income effect, which is generally thought to be the case. However, many estimates of the coefficient of relative risk aversion put it between one and two. Hence, $\rho = 0.3$ in one parameterization, and $\rho = 1.2$ in the other. In each, $\pi = 0.1$, $f(x_1) = 0.2$, $f(x_h) = 0.8$, $x_l = 0.1$, and $x_h = 1.56$.[7]

Figure 3 depicts the seigniorage Laffer curves net of the government's interest obligations for select values of μ and $\rho < 1$. When $\mu = 0$, 100 percent of seigniorage revenue is available to finance the deficit since there are no bonds on which to pay interest. Thus, the Laffer curve has its traditional shape, reflecting the dominance of the substitution effect of a change in the nominal interest rate on reserve demand. That is, for a given value of μ, as the inflation rate rises, the nominal interest rate rises as well. Because the substitution effect dominates, this reduces the reserve–deposit ratio. The rising inflation rate and falling reserve–deposit ratio give the curve its typical hump shape.

As μ increases, the real interest rate rises to restore equilibrium in the bond market. Together, the increase in the stock of bonds and the real interest rate drive up the government's interest obligations. With more seigniorage revenue going to finance the government's interest expenses, less revenue is available to finance a deficit. Thus, the maximum budget deficit consistent with equilibrium shrinks, shifting the Laffer curve down. As μ increases, the government's interest expense, which is increasing at a decreasing rate, becomes the more dominant term in the government's budget constraint. Its impact on the budget constraint ultimately distorts the shape of the curve. This is apparent in Fig. 3.

[7] The parameterizations for which results are illustrated are simply those most visually appealing in that the Laffer curve has more its textbook shape and the relevant findings are more easily seen.

Fig. 4 Welfare for $\rho = 0.3, \pi = 0.1, x_l = 0.1, x_h = 1.56$

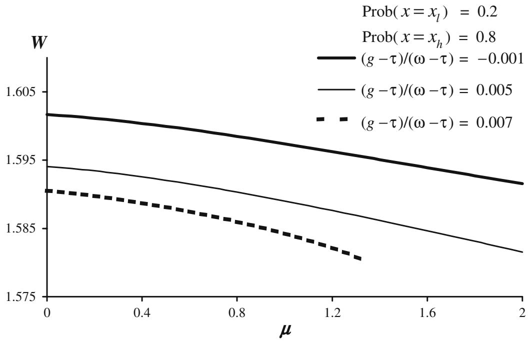


Figure 4 shows the effect of changes in μ on steady-state welfare for various values of $(g - \tau)\big/(\omega - \tau)$ and the same parameter values used to construct Fig. 3. For a given value of $(g - \tau)\big/(\omega - \tau)$, welfare is decreasing in μ on the good side of the Laffer curve for the same reasons described in the Sect. 6.1: the benefits that bonds provide in terms of insurance against the risky return on storage are offset by the costs in reduced insurance against relocation due to the higher inflation rate. Similarly, for a given μ, the bigger the primary budget deficit, the more revenue the government must raise via seigniorage. As a result, steady-state welfare decreases as the primary budget deficit increases.

Figures 5 and 6 show the seigniorage Laffer curves, net of interest obligations, and steady-state welfare for the case of $\rho > 1$. The income effect of a change in the nominal interest rate is dominant when $\rho > 1$. This means that along a Laffer curve in Fig. 5, as the inflation rate rises, the nominal interest rate rises also and drives up the reserve–deposit ratio. As a result, when $\mu = 0$, so there are no interest obligations to finance, the dominant income effect causes the Laffer curve to slope up everywhere. But as before, as μ increases, the government must use an increasing share of its seigniorage revenue to make the promised interest payments on its bonds. The Laffer curve again shifts down and its shape becomes distorted. In this case, however, it takes on a more typical shape as μ increases because the government's interest obligations, which increase with the inflation rate, drag the right end of the curve down more than the left. This allows the possibility of the economy being on the bad side of the Laffer curve when $\mu > 0$, a possibility that does not exist when $\mu = 0$.

Figure 6 shows steady-state welfare as a function of μ for the parameter values used to construct Fig. 5. Welfare is again decreasing in μ, assuming the economy is operating on the good side of the Laffer curve. This is due solely to the fact that when $R > 1$, regardless of the value of ρ, the Laffer curve shifts down as μ increases.

The results apparent in Figs. 3, 4, 5, and 6, do not hold if the economy is on the bad side of the Laffer curve. On the bad side of the Laffer curve the results are reversed, as discussed in Sect. 6.1. As μ increases, the demand for money rises, the inflation tax falls, and welfare increases.

Likewise, while attention has been restricted to steady states with $R > 1$, it is interesting to think about how the analysis would differ if focused on steady states with

Fig. 5 Seigniorage Laffer curves net of interest obligations for $\rho = 1.2, \pi = 0.1, x_l = 0.4, x_h = 5.0$

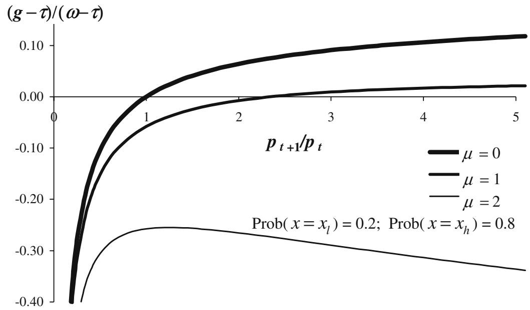
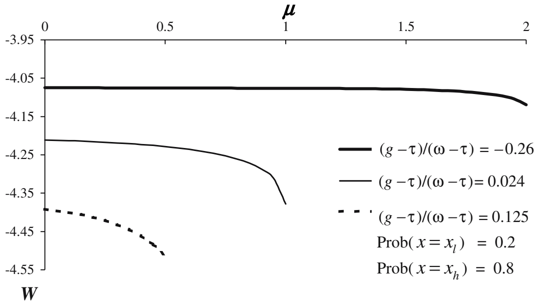


Fig. 6 Welfare for $\rho = 1.2, \pi = 0.1, x_l = 0.4, x_h = 5.0, q = 0.2$



$R < 1$. In steady states with $R < 1$ and the economy on the good side of the Laffer curve, welfare increases as μ increases because the government earns interest on its bonds. These interest earnings shift the Laffer curve up as μ increases, allowing the government to finance its expenditures with a lower inflation rate and bank deposits to offer better insurance against the risk of relocation. This is the case regardless of whether $\rho > 1$ or $\rho < 1$. The magnitude of ρ only determines whether there is a bad side of the Laffer curve.

7 Steady states with $I > 1$ and tax financing

This section considers the social value of government debt when the government must finance its obligations by varying τ, which in this context is a nondistorting tax. Under this financing scenario, the rate of money creation is held fixed as μ is varied. Only the local analysis with $\mu = 0$, as in Sect. 6.1, is presented here. The results from analysis with $\mu > 0$ are the same as those from the local analysis, as is the intuition for them, so they are not presented.

To examine the social value of risk-free government debt when interest obligations on the debt are tax financed, τ is allowed to change along with μ while the rate of money creation, and hence the steady-state inflation rate, remains constant. Under this scenario, the envelope theorem implies that

$$W'(\mu) = -(\omega - \tau)^{-\rho}\left(\frac{\partial \tau}{\partial \mu}\right)\left\{\pi\left[\left(\frac{\gamma^m}{\pi}\right)\left(\frac{p_t}{p_{t+1}}\right)\right]^{1-\rho} + (1-\pi)\int_{\underline{x}}^{\bar{x}}\left[\frac{R\gamma^b + x(1-\gamma^m-\gamma^b)}{1-\pi}\right]^{1-\rho} f(x)\mathrm{d}x\right\}$$
$$+\left(\frac{\partial R}{\partial \mu}\right)(\omega-\tau)^{1-\rho}(1-\pi)^{\rho}\gamma^b\int_{\underline{x}}^{\bar{x}}\left[R\gamma^b + x\left(1-\gamma^m-\gamma^b\right)\right]^{-\rho} f(x)\mathrm{d}x.$$

Evaluating this derivative at $\mu = 0 = \gamma^b$ gives

$$W'(0) = -(\omega - \tau)^{-\rho}\left(\frac{\partial \tau}{\partial \mu}\right)\bigg|_{\mu=0}\left\{\pi\left[\left(\frac{\gamma^m}{\pi}\right)\left(\frac{p_t}{p_{t+1}}\right)\right]^{1-\rho} + (1-\pi)\int_{\underline{x}}^{\bar{x}}\left[\frac{x(1-\gamma^m)}{1-\pi}\right]^{1-\rho} f(x)\mathrm{d}x\right\}.$$

The sign of $W'(0)$ is the opposite of the sign of $\left(\partial\tau/\partial\mu\right)\big|_{\mu=0}$. The appendix shows that the sign of $\left(\partial\tau/\partial\mu\right)\big|_{\mu=0}$ satisfies

$$\left[\frac{\omega - g}{(w-\tau)^2}\right]\frac{\partial \tau}{\partial \mu}\bigg|_{\mu=0} = (R-1)\gamma^m. \tag{27}$$

Proposition 3 follows directly:

Proposition 3 *Given* $R > 1$, $\left(\partial\tau/\partial\mu\right)\big|_{\mu=0} > 0$ *and thus* $W'(0) < 0$.

In short, if $R > 1$, and if interest payments on the government debt can be financed with nondistorting taxation, then steady-state welfare is not increased locally by raising government debt levels above zero. Any benefit bonds offer in terms of better insurance against portfolio risk is outweighed by the cost of the taxation needed to finance the government's interest payments on the bonds.

It is again interesting to think of how the results differ in economies with $R < 1$. In that case, this result is reversed, but deeper issues arise. If $R < 1$, and if τ can be varied, the best policy may be to keep $p_t/p_{t+1} \geqslant 1$, which may drive agents away from investing in risky storage altogether. Hence, this possibility is not examined further.

8 Conclusion

This paper considers whether an economy is better off with a stock of risk-free government bonds solely because the bonds allow for greater portfolio diversification and thus better insurance against risk. It finds that it is rational for banks to hold interest-bearing risk-free government bonds if they are available. But the benefits from doing so are generally outweighed by the increased cost to society arising from the higher tax rates needed to finance the government's interest payments on those bonds. Only under a number of very specific conditions—including that the economy is operating on the bad side of the Laffer curve—does the introduction of interest-bearing, risk-free bonds raise welfare. It is never optimal, however, to introduce risk-free bonds that do not bear interest (that is, the Friedman rule is not optimal).

Since markets are complete here under appropriate assumptions, these results raise the question of whether some form of market incompleteness is necessary for the availability of government bonds to raise welfare. Most of the literature on the social value of government debt has assumed incomplete markets in the form of borrowing constraints. That literature easily finds a role for government bonds in relaxing those constraints.

There is another possible role for risk-free government debt, a role not considered here. Some financial market analysts have argued that risk-free government bonds are socially beneficial because their rate of return is a risk-free rate that serves as a benchmark in pricing more risky assets. Others, however, have argued that even if there were such a benefit to having risk-free government bonds, the private sector could easily provide an asset with that attribute. This is certainly a theoretical possibility. Whether it is also a practical possibility is unclear. If it is, then there is no necessary role for the government in issuing interest-bearing, risk-free debt.

Appendix

Derivation of (24)–(26)

This appendix shows that $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}$ depends on the sign of $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}$, which in turn depends on the sign of μ and the sign of

$$H\left(\frac{p_{t+1}}{p_t}\right) \equiv (1-\rho)\left(\frac{p_{t+1}}{p_t}\right) - \rho\pi\left(\frac{p_{t+1}}{p_t}\right)^{\frac{\rho-1}{\rho}}\left[(1-\pi)^{\rho}\int\limits_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x\right]^{\frac{-1}{\rho}}.$$

Differentiating the government budget constraint (Eq. (10)) with respect to μ yields

$$\gamma^b\frac{\partial R}{\partial\mu} + (R-1)\frac{\partial\gamma^b}{\partial\mu} = \left(1-\frac{p_t}{p_{t+1}}\right)\frac{\partial\gamma^m}{\partial\mu} - \gamma^m\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}.$$

Evaluating this expression at $\mu = 0 = \gamma^b$ gives

$$\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0} = -(R-1) + \left(1-\frac{p_t}{p_{t+1}}\right)\left(\frac{1}{\gamma^m}\right)\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}, \qquad (28)$$

one equation in $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0}$ and $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}$.

Using (13), (11) can be rewritten as

$$\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^m\right)^{-\rho}=(1-\pi)^{\rho}R\int_{\underline{x}}^{\bar{x}}\left[R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)\right]^{-\rho}f(x)\mathrm{d}x. \quad (29)$$

Differentiating (13) and (29) with respect to μ yields

$$\begin{aligned}&\frac{\partial}{\partial\mu}\left\{R\int_{\underline{x}}^{\bar{x}}\left[R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)\right]^{-\rho}f(x)\mathrm{d}x\right\}\\&=-\rho\int_{\underline{x}}^{\bar{x}}\left\{x\left[(R-x)\frac{\partial\gamma^b}{\partial\mu}+\gamma^b\frac{\partial R}{\partial\mu}-x\frac{\partial\gamma^m}{\partial\mu}\right]\right.\\&\qquad\left.\times\left[R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)\right]^{-(1+\rho)}\right\}f(x)\mathrm{d}x\end{aligned} \quad (30)$$

and

$$\begin{aligned}&(1-\rho)\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\left(\gamma^m\right)^{-\rho}\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}-\rho\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^m\right)^{-(1+\rho)}\frac{\partial\gamma^m}{\partial\mu}\\&=(1-\pi)^{\rho}\frac{\partial}{\partial\mu}\left\{R\int_{\underline{x}}^{\bar{x}}\left[R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)\right]^{-\rho}f(x)\mathrm{d}x\right\}.\end{aligned} \quad (31)$$

Substituting (30) into (31) and evaluating the result at $\mu=0=\gamma^b$ gives

$$\begin{aligned}&(1-\rho)\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\left(\gamma^m\right)^{-\rho}\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}-\rho\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^m\right)^{-(1+\rho)}\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\\&=-(1-\pi)^{\rho}\rho\int_{\underline{x}}^{\bar{x}}\left\{x^{-\rho}\left(1-\gamma^m\right)^{-(1+\rho)}\left[(R-x)\gamma^m-x\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\right]\right\}f(x)\mathrm{d}x.\end{aligned} \quad (32)$$

Using the fact that, with $\mu=0=\gamma^b$, (13) becomes

$$\int_{\underline{x}}^{\bar{x}}(R-x)x^{-\rho}f(x)\mathrm{d}x=0,$$

Eq. (32) can be written (after rearranging terms) as

$$\left(\frac{1-\rho}{\rho}\right)\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\gamma^m \left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}$$
$$= \left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\left\{\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}+(1-\pi)^{\rho}\left(\frac{\gamma^m}{1-\gamma^m}\right)^{1+\rho}\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x\right\}. \quad (33)$$

Since (16) implies that

$$\left[\frac{\gamma^m}{1-\gamma^m}\right]^{1+\rho} = \pi^{1+\rho}\left\{(1-\pi)^{\rho}\left[\int_{\underline{x}}^{\bar{x}}\left[x\left(\frac{p_{t+1}}{p_t}\right)\right]^{1-\rho} f(x)\mathrm{d}x\right]\right\}^{-\left(\frac{1+\rho}{\rho}\right)}$$

when $\mu = 0$, (33) can be written as

$$\left(\frac{1-\rho}{\rho}\right)\left(\frac{p_t}{p_{t+1}}\right)^{-1}\gamma^m \left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}$$
$$= \left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\left\{1+\pi\left(\frac{p_t}{p_{t+1}}\right)^{\frac{1-\rho}{\rho}}\left[(1-\pi)^{\rho}\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x\right]^{\frac{-1}{\rho}}\right\}, \quad (34)$$

providing a second equation in $\left.\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\right|_{\mu=0}$ and $\left.\left(\partial\gamma^m/\partial\mu\right)\right|_{\mu=0}$.

Solving (28) and (34) for $\left.\left(\partial\gamma^m/\partial\mu\right)\right|_{\mu=0}$ yields

$$-\left(\frac{1-\rho}{\rho}\right)\left(\frac{p_{t+1}}{p_t}\right)\gamma^m(R-1) = \left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\left(\frac{1}{\rho}\right)\left\{1-H\left(\frac{p_{t+1}}{p_t}\right)\right\}, \quad (35)$$

where

$$H\left(\frac{p_{t+1}}{p_t}\right) \equiv (1-\rho)\left(\frac{p_{t+1}}{p_t}\right)-\rho\pi\left(\frac{p_{t+1}}{p_t}\right)^{\frac{\rho-1}{\rho}}\left[(1-\pi)^{\rho}\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x\right]^{\frac{-1}{\rho}}.$$

Derivation of (27)

Differentiating the government budget constraint (10) with respect to μ yields

$$(R-1)\frac{\partial\gamma^b}{\partial\mu}+\gamma^b\frac{\partial R}{\partial\mu}=\left(1-\frac{p_t}{p_{t+1}}\right)\frac{\partial\gamma^m}{\partial\mu}+\left[\frac{\omega-g}{(\omega-\tau)^2}\right]\frac{\partial\tau}{\partial\mu}.$$

Evaluating this expression at $\mu=0=\gamma^b$ and using $\left(\partial\gamma^b/\partial\mu\right)\big|_{\mu=0}=\gamma^m$ gives

$$(R-1)\gamma^m=\left(1-\frac{p_t}{p_{t+1}}\right)\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}+\left[\frac{\omega-g}{(\omega-\tau)^2}\right]\left.\frac{\partial\tau}{\partial\mu}\right|_{\mu=0}. \tag{36}$$

Differentiating (5) with respect to μ yields

$$\pi^\rho\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^m\right)^{-(1+\rho)}\frac{\partial\gamma^m}{\partial\mu}=(1-\pi)^\rho\int_{\underline{x}}^{\bar{x}}\left\{x\left[R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)\right]^{-(1+\rho)}\right.$$
$$\left.\times\left[(R-x)\frac{\partial\gamma^b}{\partial\mu}+\gamma^b\frac{\partial R}{\partial\mu}-x\frac{\partial\gamma^m}{\partial\mu}\right]f(x)\mathrm{d}x\right\}.$$

Evaluating this at $\mu=0=\gamma^b$ and using $\left(\partial\gamma^b/\partial\mu\right)\big|_{\mu=0}=\gamma^m$ yields

$$\begin{aligned}&\pi^\rho\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^m\right)^{-(1+\rho)}\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\\&=(1-\pi)^\rho\gamma^m\left(1-\gamma^m\right)^{-(1+\rho)}\int_{\underline{x}}^{\bar{x}}(R-x)x^{-\rho}f(x)\mathrm{d}x\\&\quad-(1-\pi)^\rho\left(1-\gamma^m\right)^{-(1+\rho)}\left(\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\right)\int_{\underline{x}}^{\bar{x}}x^{1-\rho}f(x)\mathrm{d}x.\end{aligned} \tag{37}$$

Finally, when $\mu=0=\gamma^b$, Eq. (13) implies that

$$\int_{\underline{x}}^{\bar{x}}(R-x)x^{-\rho}f(x)\mathrm{d}x=0.$$

Thus, (37) reduces to

$$\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\frac{1-\gamma^m}{\gamma^m}\right)^{(1+\rho)}\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0} = -(1-\pi)^{\rho}\left(\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\right)\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\mathrm{d}x.$$

Clearly, this condition implies that $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0} = 0$. Therefore, from (36),

$$\left[\frac{\omega-g}{(\omega-\tau)^2}\right]\left.\frac{\partial\tau}{\partial\mu}\right|_{\mu=0} = (R-1)\gamma^m.$$

References

Aiyagari, S.R., McGrattan, E.R.: The optimum quantity of debt. J Monet Econ **42**, 447–469 (1998)

Aiyagari, S.R., Gertler, M.: Asset returns with transactions costs and uninsured individual risk. J Monet Econ **27**, 311–331 (1991)

Barro, R.J.: Optimal management of indexed and nominal debt. NBER Working Paper No. 6197 (1997)

Bhattacharya, J., Guzman, M.G., Smith, B.D.: Some even more unpleasant monetarist arithmetic. Can J Econ **31**, 596–623 (1998)

Cavalcanti, T., Villamil, A.: The optimal inflation tax and structural reform. Macroecon Dyn **7**, 333–362 (2003)

Chari, V.V., Kehoe, P.J.: Optimal fiscal and monetary policy. In: Taylor, J.B., Woodford, M. (eds.) Handbook of Macroeconomics, Vol. 1. Elsevier Science, New York (1999)

Diamond, D.W., Dybvig, P.H.: Bank runs, deposit insurance, and liquidity. J Polit Econ **91**, 401–419 (1983)

Hall, G.J., Krieger, S.: The tax smoothing implications of the federal debt paydown. Brookings Papers Econ Activity **2**, 253–284 (2000)

Kocherlakota, N.R.: Societal benefits of illiquid bonds. J Econ Theory **108**, 179–193 (2003)

Lucas, R.E., Jr., Stokey, N.L.: Optimal fiscal and monetary policy in an economy without capital. J Monet Econ **12**, 55–93 (1983)

Paal, B., Smith, B.D.: The sub-optimality of the Friedman rule, and the optimum quantity of money. Unpublished manuscript (2000)

Romer, D.: Why should governments issue bonds? J Money Credit Bank **25**, 163–175 (1993)

Sargent, T.J., Wallace, N.: Some unpleasant monetarist arithmetic. Feder. Reserv. Bank Minneap. Quart Rev **9**, 15–31 (1981)

Schreft, S.L., Smith, B.D.: The conduct of monetary policy with a shrinking stock of government debt. J Money Credit Bank **34**, 848–882 (2002)

Schreft, S.L., Smith, B.D.: The evolution of cash transactions: some implications for monetary policy. J Monet Econ **46**, 97–120 (2000)

Smith, B.: Taking intermediation seriously. J Money Credit Bank **35**, 1319–1357 (2003)

Smith, B., Villamil, A.: Government borrowing using bonds with randomly determined return: welfare improving randomization in the context of deficit finance. J Monet Econ **41**, 351–370 (1998)

Townsend, R.M.: Models of money with spatially separated agents. In: Kareken, J.H., Wallace, N. (eds.) Models of monetary economies. Federal Reserve Bank of Minneapolis, Minneapolis (1980)

Townsend, R.M.: Economic organization with limited communication. Am Econ Rev **77**, 954–971 (1987)

Villamil, A.P.: Price discriminating monetary policy: a nonuniform pricing approach. J Public Econ **35**, 385–392 (1988)

Woodford, M.: Public debt as private liquidity. Am Econ Rev **80**, 382–388 (1990)